

# Quasi-alternating Montesinos links

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## Abstract

The aim of this article is to detect new classes of quasi-alternating links. Quasi-alternating links are a natural generalization of alternating links. Their knot Floer and Khovanov homology are particularly easy to compute. Since knot Floer homology detects the genus of a knot as well as whether a knot is fibered, as provided bounds on unknotting number and slice genus, characterization of quasi-alternating links becomes an interesting open problem. We show that there exist classes of non-alternating Montesinos links, which are quasi-alternating.

## 1 Introduction

Quasi-alternating links were introduced by Ozsvath and Szabo [14]. It was shown in [13] that their knot Floer homology can be computed explicitly and depends only on the signature and the Alexander polynomial of the knot. More precisely it was shown that quasi-alternating links are homologically thin for both Khovanov homology and knot Floer homology. The definition is given in a recursive way:

**Definition 1.1** ([14]). The set  $\mathcal{Q}$  of *quasi-alternating links* is the smallest set of links which satisfies the following properties:

- The unknot is in  $\mathcal{Q}$ .
- If the link  $L$  has a diagram with a crossing  $c$  such that
  - (i) both smoothings of  $c$ ,  $L_0$  and  $L_\infty$  as in Figure 1, are in  $\mathcal{Q}$ ,
  - (ii)  $\det(L_0), \det(L_\infty) \neq 0$ ,
  - (iii)  $\det(L) = \det(L_0) + \det(L_\infty)$ ;

then  $L$  is in  $\mathcal{Q}$ . The crossing  $c$  is called a *quasi-alternating crossing* of  $L$  and  $L$  is called *quasi-alternating at  $c$* .

The class of quasi-alternating links contains all alternating links [14]. It was shown by Champanerkar and Kofman [4] that the sum of two quasi-alternating links is quasi-alternating and that a quasi-alternating crossing can be replaced

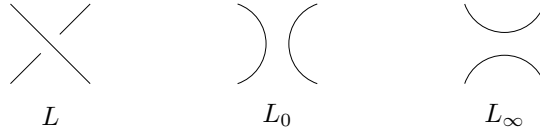


Figure 1: The link  $L$  at crossing  $c$  and its resolutions  $L_0$  and  $L_\infty$ .

by an alternating rational tangle to obtain another quasi-alternating link. Moreover they applied this result to show that there exist a family of pretzel links which is quasi-alternating. We will apply their method to Montesinos links.

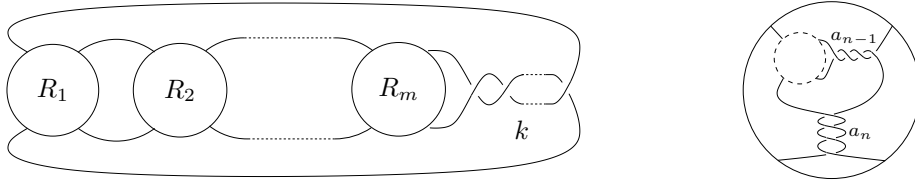


Figure 2: Montesinos link.

A Montesinos link admits a diagram  $D$  composed of  $m \in \mathbb{N}$  rational tangle diagrams  $R_1, R_2, \dots, R_m$  and  $k \in \mathbb{N}_0$  half-twists put together as in Figure 2. We will denote such a link by  $L(R_1, R_2, \dots, R_m; k)$ . The rational tangles can be obtained from a sequence of non-zero integers  $a_1, a_2, \dots, a_n$  as indicated in Figure 2, and they are denoted by  $R = a_1 a_2 \cdots a_n$ . Our goal will be to prove the following theorem.

**Theorem 1.2.** *Let  $R = b_1 b_2 \cdots b_m$  represent a rational tangle with at least two crossings and let  $a_i, b_i, c_i, m, n \in \mathbb{N}$  for all  $i$  and  $n \geq 2$ . Then the following three Montesinos links yield infinite families of non-alternating, quasi-alternating links:*

- (i)  $L(a_1 a_2, R, -n)$  with  $1 + a_1(a_2 - n) < 0$ ,
- (ii)  $L(a_1 a_2, R, (-c_1)(-c_2))$  with  $a_2 < c_2$  or  $a_2 = c_2$  and  $a_1 > c_1$ ,
- (iii)  $L(a_1 a_2 a_3, R, -n)$  with  $a_3 < n$ .

## 2 Determinant

The determinant of an alternating link is related to the number of spanning trees of its checkerboard graph. We will apply a generalization of this result obtained by Dasbach, Futer, Kalfagianni, Lin, and Stoltzfus [5] to compute the determinant of Montesinos links. This for we recall the following definitions.

**Definition 2.1** ([11]). The *all-A dessin*  $\mathbb{D}(A)$  of a link, also known as *ribbon graph*, is a graph which can be constructed out of a link diagram in the following way:

First, each crossing is replaced by an A-splicing (see Figure 3). This results in a collection of circles in the plane with line segments joining them. Out of this projection, the all-A dessin is obtained by contracting each circle to a point such that the vertices of  $\mathbb{D}(A)$  are in one-to-one correspondence with the circles. The edges then correspond to the line segments between them. The construction of the *all-B dessin*  $\mathbb{D}(B)$  can be done analogously by replacing each crossing by a B-splicing.

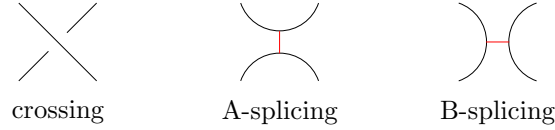


Figure 3: Splicings of a crossing.

If  $\mathbb{D}$  is a dessin of a link  $L$  there exists an orientation on it such that  $\mathbb{D}$  can be viewed as a multi-graph equipped with a cyclic order on the edges at every vertex (for the exact construction see [11]). Therefore the dessin corresponds to a graph embedded on an orientable surface such that every region in the complement of the graph is a disc. We call the regions the *faces* of the dessin.

**Definition 2.2** ([11]). Let  $\mathbb{D}$  be a dessin with one connected component and denote by  $v(\mathbb{D})$ ,  $e(\mathbb{D})$  and  $f(\mathbb{D})$  the number of vertices, edges and faces in  $\mathbb{D}$ . The *dessin genus* is calculated as follows:

$$g(\mathbb{D}(A)) = \frac{2 - (v(\mathbb{D}) - e(\mathbb{D}) + f(\mathbb{D}))}{2}.$$

To compute  $f(\mathbb{D}(A))$  the fact that  $\mathbb{D}(A)$  and  $\mathbb{D}(B)$  are dual to each other [11, 6] is used. Hence  $f(\mathbb{D}(A)) = v(\mathbb{D}(B))$ . Given these definitions a generalized formula to calculate the determinant of links with an all-A dessin of genus one can be stated.

**Theorem 2.3** ([5]). Let  $\mathbb{D}(A)$  and  $\mathbb{D}(B)$  be the all-A respectively the all-B dessins of a connected link projection of a link  $L$ . Suppose  $\mathbb{D}(A)$  is of dessin genus one. Then

$$\det(L) = |\#\{\text{spanning trees in } \mathbb{D}(A)\} - \#\{\text{spanning trees in } \mathbb{D}(B)\}|.$$

**Lemma 2.4.** The dessin genus of a non-alternating Montesinos diagram equals one.

*Proof.* A non-alternating Montesinos link diagram can be obtained out of a non-alternating pretzel link diagram  $P(p_1, \dots, p_n, -q_1, \dots, -q_m)$  by replacing

the tassels with rational tangles. Inserting a rational tangle does not change the dessin genus, therefore the Montesinos link will have the same dessin genus as the pretzel link. It was shown in [4] that this pretzel link diagrams have dessin genus one, thus the non-alternating Montesinos diagrams have dessin genus one too.  $\square$

According to this lemma the above theorem can be applied to Montesinos links. Moreover it shows that the Turaev genus of a non-alternating Montesinos link equals one.

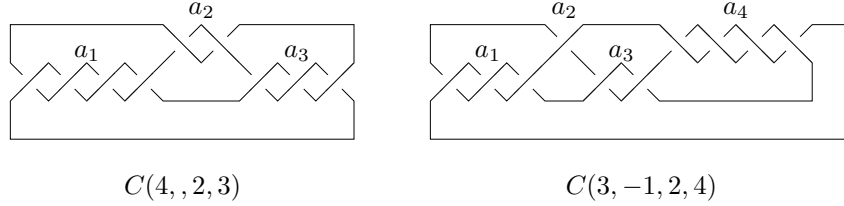


Figure 4: Examples of rational links.

A rational link is a link which admits a projection as in Figure 4 and it is denoted by  $C(a_1, a_2, \dots, a_n)$ . The determinant of these links has been studied by Kauffman and Lopes [8].

**Lemma 2.5** ([8]). *Let  $n_i \in \mathbb{Z} \setminus \{0\}$  then we have*

$$\begin{aligned}
 \det C(n_1) &= |n_1| \\
 \det C(n_1, n_2) &= |1 + n_1 n_2| \\
 \det C(n_1, n_2, n_3) &= |n_1 + n_3 + n_1 n_2 n_3| \\
 \det C(n_1, n_2, n_3, n_4) &= |1 + n_1 n_2 + n_1 n_4 + n_3 n_4 + n_1 n_2 n_3 n_4|.
 \end{aligned}$$

### 3 Quasi-alternating Montesinos links

To obtain a family of non-alternating Montesinos links we will need the following definition.

**Definition 3.1** ([10]). A diagram  $D$  as in Figure 2 is called a *reduced Montesinos diagram* if it satisfies one of the following two conditions:

- (i)  $D$  is alternating, or
- (ii) Each  $R_i$  is an alternating rational tangle diagram with at least two crossings placed in  $D$  such that the two lower ends of  $R_i$  belong to arcs incident to a common crossing and  $k = 0$ .

It was shown by Lickorish and Thistlethwaite [10] that a link which admits a  $n$ -crossing, reduced Montesinos diagram cannot be projected with fewer than  $n$  crossings.

**Lemma 3.2.** *A link which admits a non-alternating reduced Montesinos diagram is non-alternating.*

*Proof.* Let  $L$  be a link which admits a non-alternating, reduced Montesinos diagram with  $n$  crossings. Therefore the minimal crossing number has to be  $n$ . Assume  $L$  is alternating. Then  $L$  possesses a connected, reduced, alternating diagram with  $m$  crossings. According to a lemma of Lickorish [9]  $m$  is strictly smaller than any crossing number of a non-alternating diagram of the same link. Therefore  $m < n$  which is a contradiction.  $\square$

**Proof of Theorem 1.2.** Let  $R = b_1 b_2 \cdots b_m$  represent a rational tangle with at least two crossings and let  $a_i, b_i, c_i, m, n \in \mathbb{N}$  for all  $i$  and  $n \geq 2$ . Further let  $L^1 = L(a_1 a_2, R, -n)$ ,  $L^2 = L(a_1 a_2, R, (-c_1)(-c_2))$  and  $L^3 = L(a_1 a_2 a_3, R, -n)$ . The links  $L^1, L^2$  and  $L^3$  are non-alternating since they possess a non-alternating, reduced Montesinos link diagram.

Now let  $\widehat{L^i}$  be the link  $L^i$  with the rational tangle  $R$  replaced by one single positive crossing called  $c$ , for  $i \in \{1, 2, 3\}$ . First we show that the resolutions  $\widehat{L_0^i}$  and  $\widehat{L_\infty^i}$  are quasi-alternating at  $c$ . For all  $i$ , the link  $\widehat{L_0^i}$  is the sum of two alternating links and therefore quasi-alternating.

The resolutions  $\widehat{L_\infty^i}$  are rational links:

$$\begin{aligned}\widehat{L_\infty^1} &= C(a_1, (a_2 - n)) \\ \widehat{L_\infty^2} &= C(a_1, (a_2 - c_2), -c_1) \\ \widehat{L_\infty^3} &= C(a_1, a_2, (a_3 - n)).\end{aligned}$$

Since each rational link possesses an alternating projection (see Bankwitz and Schumann [2]), the resolutions  $\widehat{L_\infty^i}$  are quasi-alternating. It remains to show that the determinants add up correctly.

- (i) For  $\widehat{L^1}$  let  $1 + a_1(a_2 - n) < 0$ . By applying Lemma 2.5, the determinants of the resolutions for  $\widehat{L^1}$  at  $c$  hold:

$$\begin{aligned}\det(\widehat{L_0^1}) &= \det(T(2, -n) \# C(a_1, a_2)) = \det T(2, -n) \cdot \det C(a_1, a_2) \\ &= n(1 + a_1 a_2) \\ \det(\widehat{L_\infty^1}) &= \det C(a_1, (a_2 - n)) = |1 + a_1(a_2 - n)| \\ &= (-1)(1 + a_1(a_2 - n)).\end{aligned}$$

The determinant for  $\widehat{L^1}$  can be calculated out of the diagrams of its all-A and all-B dessins. The number of spanning trees can be computed directly by inspecting the diagrams of Figure 5:

$$\begin{aligned}\#\{\text{spanning trees in } \mathbb{D}(A)\} &= n(a_1(a_2 + 1) + 1) \\ \#\{\text{spanning trees in } \mathbb{D}(B)\} &= a_1 a_2 + 1.\end{aligned}$$

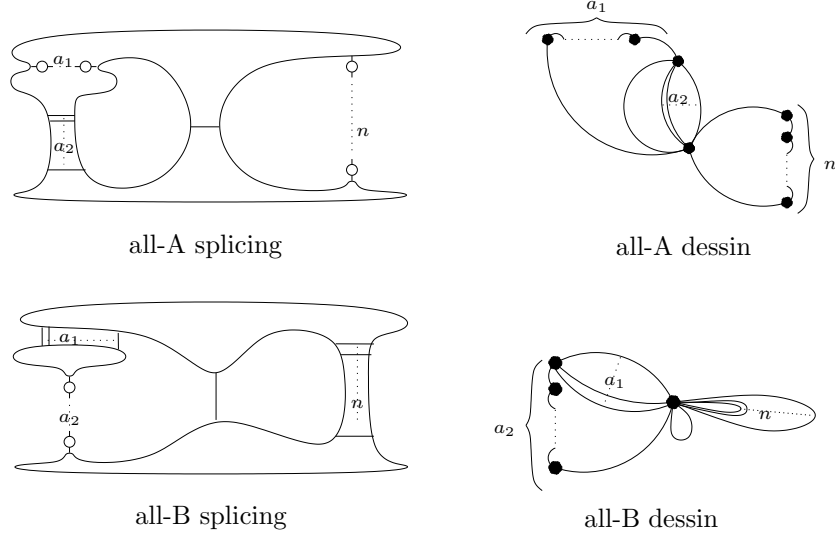


Figure 5: All-A/B splittings and the all-A/B dessins of  $\widehat{L}^1 = L(a_1 a_2, 1, -n)$ .

By Theorem 2.3 we get  $\det(L) = |n(a_1 a_2 + a_1 + 1) - a_1 a_2 - 1|$ . Hence

$$\begin{aligned}
 \det(L_0) + \det(L_\infty) &= \underbrace{n(1 + a_1 a_2)}_{>0} + \underbrace{(-1)(1 + a_1(a_2 - n))}_{>0} \\
 &= |n(a_1 a_2 + a_1 + 1) - a_1 a_2 - 1| \\
 &= \det(L).
 \end{aligned}$$

(ii) For  $\widehat{L}^2$  let  $a_2 < c_2$  or  $a_2 = c_2$  and  $a_1 > c_1$ . For the determinants of the resolutions we get:

$$\begin{aligned}
 \det(\widehat{L}_0^2) &= \det(C(a_1, a_2) \# C(c_1, c_2)) = \det C(a_1, a_2) \cdot \det C(c_1, c_2) \\
 &= (1 + a_1 a_2)(1 + c_1 c_2) \\
 \det(\widehat{L}_\infty^2) &= |a_1 - c_1 - a_1 c_1 \underbrace{(a_2 - c_2)}_{\leq 0}| \\
 &= a_1 - c_1 - a_1 c_1 (a_2 - c_2) \\
 &= a_1(1 + c_1 c_2) - c_1(1 + a_1 a_2).
 \end{aligned}$$

The number of spanning trees of the all-A/B dessins of  $\widehat{L}^2$  (see Figure 6) is given by:

$$\begin{aligned}
 \#\{\text{spanning trees in } \mathbb{D}(A)\} &= (a_1 a_2 + a_1 + 1)(c_1 c_2 + 1) \\
 \#\{\text{spanning trees in } \mathbb{D}(B)\} &= c_1(a_1 a_2 + 1).
 \end{aligned}$$

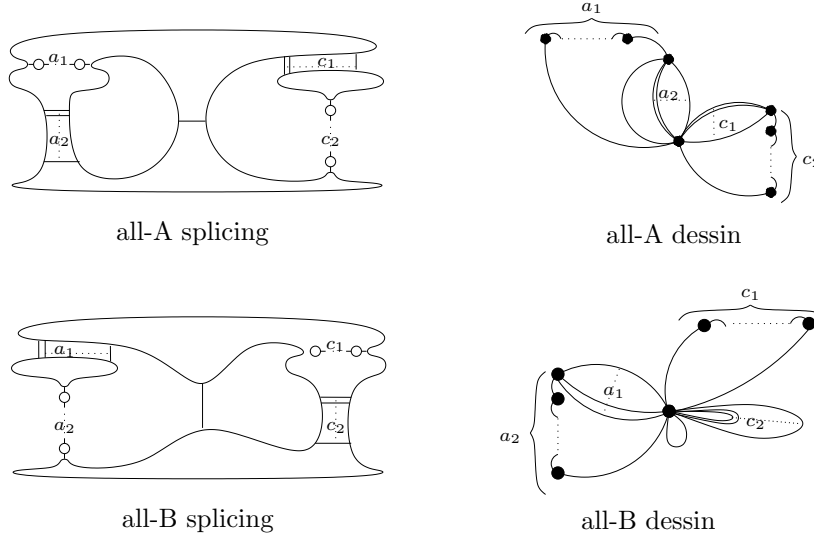


Figure 6: All-A/B splicings and the all-A/B dessins of  $\widehat{L^2} = L(a_1a_2, 1, (-c_1)(-c_2))$ .

Now according to Theorem 2.3 we get:

$$\begin{aligned}
 \det(L) &= |(a_1a_2 + a_1 + 1)(c_1c_2 + 1) - c_1(a_1a_2 + 1)| \\
 &= |(a_1a_2 + 1)(c_1c_2 + 1) + a_1(c_1c_2 + 1) - c_1(a_1a_2 + 1)| \\
 &= |(a_1a_2 + 1)(c_1c_2 + 1)| + |a_1(c_1c_2 + 1) - c_1(a_1a_2 + 1)| \\
 &= \det(L_0) + \det(L_\infty).
 \end{aligned}$$

(iii) For  $\widehat{L^3}$  let  $a_3 < n$ . For the determinants of the resolutions we get:

$$\begin{aligned}
 \det(\widehat{L_0^3}) &= \det(C(a_1, a_2, a_3) \# T(2, -n)) = n(a_1 + a_3 + a_1a_2a_3) \\
 \det(\widehat{L_\infty^3}) &= \det C(a_1, a_2, a_3 - n) = |a_1 + (a_3 - n) + a_1a_2(a_3 - n)| \\
 &= |a_1 + \underbrace{(a_3 - n)}_{<0} (1 + a_1a_2)| \\
 &= (-1)(a_1 + a_3 + a_1a_2a_3 - n(1 + a_1a_2)).
 \end{aligned}$$

The number of spanning trees of the all -A/B dessin of  $\widehat{L^3}$  (see Figure 7) is given by:

$$\begin{aligned}
 \#\{\text{spanning trees in } \mathbb{D}(A)\} &= n(1 + a_1 + a_3 + a_1a_2 + a_1a_2a_3) \\
 \#\{\text{spanning trees in } \mathbb{D}(B)\} &= a_1 + a_3 + a_1a_2a_3,
 \end{aligned}$$

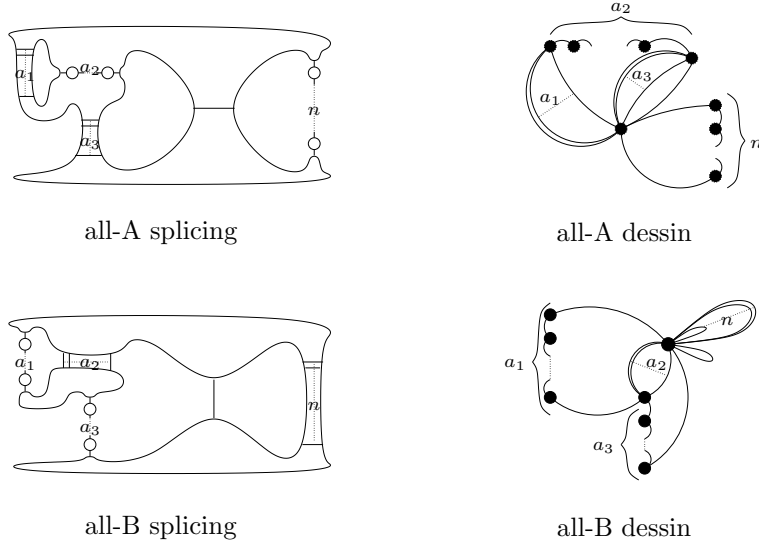


Figure 7: All-A/B splittings and the all-A/B dessins of  $\widehat{L^3} = L(a_1a_2a_3, 1, -n)$ .

which leads to

$$\begin{aligned}
\det(L) &= |n(1 + a_1 + a_3 + a_1a_2 + a_1a_2a_3) - (a_1 + a_3 + a_1a_2a_3)| \\
&= \underbrace{|n(a_1 + a_3 + a_1a_2a_3)|}_{>0} + \underbrace{|(-1)(a_1 + a_3 + a_1a_2a_3 - n(1 + a_1a_2))|}_{>0} \\
&= \det(L_0) + \det(L_\infty).
\end{aligned}$$

This shows that the determinants add up correctly for all three links  $\widehat{L^i}$ . Since all the resolutions are quasi-alternating the crossing at  $c$  is quasi-alternating. Therefore, according to [4], it can be replaced by any alternating rational tangle which extends  $c$ . This completes the proof.  $\square$

**Remark.** For  $a_i, b_i, c_i \in \mathbb{N}$  and  $R = b_1b_2 \cdots b_m$  it can be shown that the link  $L(a_1a_2a_3, R, (-c_1)(-c_2)(-c_3))$  with  $\frac{1+a_1a_2}{1+c_1c_2} > \frac{a_1+a_3+a_1a_2a_3}{c_1+c_3+c_1c_2c_3}$  is quasi-alternating. The proof is analogous to the previous one only it has to be taken in account that the sum of two rational links is always a rational link whose determinant can be calculated out of the continuous fraction which is defined by the link [7]. It is notable that the calculation of the spanning trees gets more complicated the more tassels the link has.

## 4 Examples

We will now apply Theorem 1.2 to analyze knots with 11 crossings. There exist 185 non-alternating prime knots with crossing number 11. Out of these,



67 are Montesinos links. By our method, we can identify 23 non-alternating Montesinos links to be quasi-alternating. These are listed in Table 1, together with their Conway notation according to KnotInfo [3]. Note that the minus at the last rational tangle represents a negative crossing. We have the following identities:  $[2-] = [2, -1] = [-2]$ ,  $[21-] = [21, -1] = [-3]$  and  $[3-] = [3, -1] = [(-2)(-1)]$ .

Table 1: Non-alternating, quasi-alternating Montesinos knots with 11 crossings detected to be quasi-alternating by Theorem 1.2.

Knot	Conway notation	Knot	Conway notation
11n2	$[221;211;2-]$	11n84	$[22;22;21-]$
11n3	$[221;22;2-]$	11n87	$[212;21;21-]$
11n14	$[41;211;2-]$	11n89	$[31;211;21-]$
11n15	$[41;22;2-]$	11n90	$[31;211;3-]$
11n17	$[311;211;2-]$	11n100	$[221;3;21-]$
11n18	$[311;22;2-]$	11n103	$[211;211;21-]$
11n29	$[231;21;2-]$	11n106	$[212;3;21-]$
11n30	$[231;3;2-]$	11n122	$[32;3;21-]$
11n48	$[31;22;3-]$	11n137	$[311;21;21-]$
11n63	$[411;21;2-]$	11n140	$[41;21;21-]$
11n64	$[411;3;2-]$	11n141	$[41;3;3-]$
11n83	$[31;22;21-]$		

By applying the findings of Champanerker and Kofman [4], we can identify 17 more Montesinos links to be quasi-alternating. In Table 2 for each knot there is one rational tangle of the Conway notation indicated in bold. This tangle is replaced with a crossing of the same sign and checked if it is a quasi-alternating crossing in the new diagram.

For the sake of completeness, Table 3 gives a list of all non-alternating, quasi-alternating knots up to 10 crossings detected by Manolescu [12], Baldwin [1] and Champanerker and Kofman [4]. The 16 knots indicated with a cross could also be detected to be quasi-alternating by Theorem 1.2. The remaining non-alternating knots are Khovanov homologically thick except for  $9_{46}$  and  $10_{140}$ , which are not quasi-alternating by forthcoming work of Shumakovitch.

## 4.1 Acknowledgements

I am deeply grateful to Anna Beliakova, whose help was essential for me to develop the presented results.

Table 2: Non-alternating, quasi-alternating Montesinos knots with 11 crossings detected to be quasi-alternating by inserting a rational tanlge.

Knot	Conway notation	Knot	Conway notation
11n1	<b>[23;211;2-]</b>	11n58	<b>[312;21;2-]</b>
11n13	<b>[5;211;2-]</b>	11n59	<b>[3111;21;2-]</b>
11n16	<b>[32;211;2-]</b>	11n60	<b>[3111;3;2-]</b>
11n28	<b>[24;21;2-]</b>	11n62	<b>[42;21;2-]</b>
11n51	<b>[213;21;2-]</b>	11n82	<b>[4;22;21-]</b>
11n52	<b>[2121;21;2-]</b>	11n91	<b>[4;211;21-]</b>
11n54	<b>[2112;21;2-]</b>	11n101	<b>[23;21;21-]</b>
11n55	<b>[21111;21;2-]</b>	11n105	<b>[2111;21;21-]</b>
11n56	<b>[21111;3;2-]</b>		

Table 3: Non-alternating, quasi-alternating knots up to 10 crossings.

8 <sub>20</sub>	[1]	8 <sub>21</sub>	[12]	x	9 <sub>43</sub>	[12]	x	9 <sub>44</sub>	[12]	
9 <sub>45</sub>	[12]	x	9 <sub>47</sub>	[12]		9 <sub>48</sub>	[12]	x	9 <sub>49</sub>	[12]
10 <sub>125</sub>	[1]		10 <sub>126</sub>	[1]	x	10 <sub>127</sub>	[1]	x	10 <sub>129</sub>	[4]
10 <sub>130</sub>	[4]	x	10 <sub>131</sub>	[4]	x	10 <sub>133</sub>	[4]		10 <sub>134</sub>	[4] x
10 <sub>135</sub>	[4]	x	10 <sub>137</sub>	[4]		10 <sub>138</sub>	[4]	x	10 <sub>141</sub>	[1]
10 <sub>142</sub>	[4]	x	10 <sub>143</sub>	[1]	x	10 <sub>144</sub>	[4]	x	10 <sub>146</sub>	[4] x
10 <sub>147</sub>	[4]	x	10 <sub>148</sub>	[1]		10 <sub>149</sub>	[1]		10 <sub>150</sub>	[4]
10 <sub>151</sub>	[4]		10 <sub>155</sub>	[1]		10 <sub>156</sub>	[4]		10 <sub>157</sub>	[1]
10 <sub>158</sub>	[4]		10 <sub>159</sub>	[1]		10 <sub>160</sub>	[4]		10 <sub>163</sub>	[4]
10 <sub>164</sub>	[4]		10 <sub>165</sub>	[4]		10 <sub>166</sub>	[4]			

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